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$B \to \gamma e \nu$ transitions from QCD sum rules on the light-cone

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Abstract: $B \to \gamma e \nu$ transitions have recently been studied in the framework of QCD factorization. The attractiveness of this channel for such an analysis lies in the fact that, at least in the heavy quark limit, the only hadron involved is the B meson itself, so one expects a very simple description of the form factor in terms of a convolution of the B meson distribution amplitude with a perturbative kernel. This description, however, does not include contributions suppressed by powers of the b quark mass. In this letter, we calculate corrections to the factorized expression which are induced by the "soft" hadronic component of the photon. We demonstrate that the power-suppression of these terms is numerically not effective for physical values of the b quark mass and that they increase the form factor by about 30% at zero momentum transfer. We also derive a sum rule for λ_B, the first negative moment of the B meson distribution amplitude, and find $\lambda_B = 0.6\,\mathrm{GeV}$ (to leading order in QCD).

Keywords: B-Physics, Heavy Quarks Physics, QCD, Sum Rules.

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1. Introduction

With the B factories BaBar and Belle running "full steam", B physics has entered the era of precision measurements. The quality and precision of experimental data calls for a corresponding match in theoretical precision, in particular with regard to the analysis of nonleptonic decays. Notwithstanding the fact that a complete solution of the problem appears as elusive as ever, progress has been made in the description of nonleptonic B decays, in the heavy quark limit, which were shown to be amenable to perturbative QCD (pQCD) factorization [1, 2]. In this framework, B decay amplitudes are decomposed into a "soft" part, for instance a weak decay form factor, or, in general, an intrinsically nonperturbative quantity that evades further breakdown into factorizable components, and a "hard" part which can be neatly described in a factorized form in terms of a convolution of a hard perturbative kernel, depending only on collinear momenta, with one or more hadron distribution amplitudes that describe the (collinear) momentum distribution of partons inside the hadron. Factorization was shown to hold, for certain B decay channels, to all orders in perturbation theory [3], but to date could not be extended to include contributions that are suppressed by powers of the b quark mass. For the important channel $B \to K\pi$, in particular, it was found that factorization breaks down for one specific class of power-suppressed corrections which are expected to be numerically relevant [2]. Reliable alternative methods for calculating QCD effects in nonleptonic B decays are scarce, and only little is known about the generic size of power-suppressed corrections for instance in $B \to \pi\pi$ [4], one of the "benchmark" channels for measuring the angle α of the CKM unitarity triangle. There is, however, one channel that can be treated both in pQCD factorization and by an alternative method, QCD sum rules: the $B \to \gamma\ell\nu_\ell$ transition. It has been shown recently [5, 6] that $B \to \gamma$ is indeed accessible to collinear factorization, in contrast to previous findings [7] which indicated the need to include also transverse degrees of freedom in the convolution. On the other hand, the $B \to \gamma$ transition can also been investigated in the framework of QCD sum rules on the light-cone [8, 9]. The crucial point here is that the photon is not treated as an exactly pointlike object with standard EM couplings, but that, in addition to that "hard" component, it also features a "soft" hadronic component whose contribution to the decay amplitude must not be neglected. The "soft" component is related to the probability of a real photon to dissociate, at small transverse separation, into partons and resembles in many ways a (massless) transversely polarized vector meson; like it, it can be described by a Fock-state expansion in terms of distribution amplitudes of increasing twist. An analysis of these distribution amplitudes, including terms up to twist-4, has recently been completed [10] and comes in handy for an update and extension of the previous QCD sum rules analyses of $B \to \gamma$. Such a reanalysis is the subject of this letter and we include in particular one-loop radiative corrections to the contribution of leading twist-2 distribution amplitude to $B \to \gamma$. In the framework of pQCD factorization, the "soft" component of the photon leads to formally power-suppressed contributions and is hence neglected in refs. [5, 6, 7]. As we shall show in this letter, this suppression is not effective numerically due to the large value of the matrix-element governing its strength: the magnetic susceptibility of the quark condensate. Our findings indicate that such contributions are likely to be nonneglible also in other channels involving photon emission, notably $B \to K^*\gamma$ and $B \to \rho\gamma$, which are treated in [11, 12]. We also compare the QCD sum rule for the hard part of the $B \to \gamma$ amplitude with the pQCD result and derive a sum rule for λ_B, the first negative moment of the B meson distribution amplitude.

2. Definition of relevant quantities and outline of calculation

First of all, let us define the form factors that describe the $B \to \gamma$ transition:

$$\frac{1}{\sqrt{4\pi\alpha}}\,\langle\gamma(\epsilon^*,q)|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-(p_B)\rangle = -F_V\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}v^\rho q^\sigma + iF_A[\epsilon^*_\mu(v\cdot q)-q_\mu(\epsilon^*\cdot v)]\,, \quad (2.1)$$

where ϵ^* and q are the polarisation and momentum vector of the photon, respectively, and $v = p_B/m_B$ is the four-velocity of the B meson. The definition of the form factors $F_{A,V}$ in (2.1) is exactly the same as in ref. [5].[1] The form factors depend on $p^2 = (p_B - q)^2$ or, equivalently, on the photon energy $E_\gamma = (m_B^2 - p^2)/(2m_B)$. p^2 varies in the physical region $0 \leq p^2 \leq m_B^2$, which corresponds to $0 \leq E_\gamma \leq m_B/2$.

The starting point for the calculation of the form factors from QCD sum rules is the correlation function

$$\begin{aligned}\Pi_\mu(p,q) &= i\int d^4x e^{ipx}\frac{1}{\sqrt{4\pi\alpha}}\langle\gamma(\epsilon^*,q)|T\{\bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)\bar{b}(0)i\gamma_5 u(0)\}|0\rangle \\ &= -\Pi_V\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma + i\Pi_A[\epsilon^*_\mu(p\cdot q) - q_\mu(\epsilon^*\cdot p)] + \cdots\,. \end{aligned} \quad (2.2)$$

The dots refer to contact terms which appear for pointlike photons and for a discussion of which we refer to ref. [13]; the treatment of the soft component of the photon involves non-local operators, as we shall discuss below, and gauge-invariance of Π_μ is realized explicitly, without contact terms, by working in the background field method.

[1] The difference in sign in the 1st term on the r.h.s. is due to our conventions for the epsilon tensor: we define $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5] = 4i\epsilon_{\mu\nu\rho\sigma}$, in contrast to [5], where a different sign was chosen.

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The method of QCD sum rules [14] exploits the fact that the correlation function contains information on the form factors in question: expressing $\Pi_{V(A)}$ via a dispersion relation, one has

$$\Pi_{V(A)} = \frac{f_B m_B F_{V(A)}}{m_b(m_B^2 - p_B^2)} q + \int_{s_0}^{\infty} \frac{ds}{s - p_B^2}\, \rho_{V(A)}(s, p^2)\,, \tag{2.3}$$

where the first term on the r.h.s. is the contribution of the ground state B meson to the correlation function, featuring the form factors we want to calculate, and the second term includes all other states coupling to the pseudoscalar current $\bar{b} i\gamma_5 u$, above the threshold s_0. f_B is the decay constant of the B and defined as $\langle B|\bar{b} i\gamma_5 u|0\rangle = f_B m_B^2/m_b$. The form factors $F_{A,V}$ are obtained, in principle, by equating the dispersion relation (2.3) to $\Pi_{A,V}$ calculated for Euclidean p_B^2 by means of an operator product expansion. The sum over higher states, the 2nd term on the r.h.s. of (2.3), is evaluated using quark-hadron duality, which means that the hadronic spectral density $\rho_{V(A)}$ is replaced by its perturbative equivalent. In order to reduce the model-dependence associated with that procedure, one subjects the whole expression to a Borel-transformation, which results in an exponential suppression of the continuum of higher states:

$$B_{M^2}\Pi_{V(A)} = \frac{f_B m_B F_{V(A)}}{m_b}\, e^{-m_B^2/M^2} + \int_{s_0}^{\infty} ds\, \rho_{V(A)}^{\rm pert}(s) e^{-s/M^2}\,. \tag{2.4}$$

The relevant parameters of the sum rule are then M^2, the Borel parameter, and s_0, the continuum threshold, and our results for the form factors will depend (moderately) on these parameters.

As already mentioned in the introduction, the photon does not only have EM pointlike couplings to the quarks, but also soft nonlocal ones, and we write $F_{A,V} = F_{A,V}^{\rm hard} + F_{A,V}^{\rm soft}$. The separation between these two components is of course scheme-dependent and we define them in the $\overline{\rm MS}$ scheme. For the pointlike component of the photon, Π_μ is just a three-point correlation function, described by the triangle diagrams shown in figure 1. These diagrams, and also the leading nonperturbative correction which is proportional to the quark condensate, have been calculated in refs. [8, 9], and we confirm the results; we will come back to the hard contributions in section 4. The calculation of radiative corrections to the three-point function, although highly desirable, is beyond the scope of this letter whose main emphasis is on calculating the soft photon contributions which become relevant for a certain configuration of virtualities, namely $m_b^2 - p_B^2 \geq O(\Lambda_{\rm QCD} m_b)$ and $m_b^2 - p^2 \geq O(\Lambda_{\rm QCD} m_b)$, i.e. $E_\gamma \gg \Lambda_{\rm QCD}$. In this regime, the integral in Eq. (2.3) is dominated by light-like distances and can be expanded around the light-cone:

$$\Pi_{V(A)}(p_B^2, E_\gamma) = Q_u \chi(\mu_F)\langle \bar{u}u\rangle(\mu_F) \sum_n \int_0^1 du\, \phi^{(n)}(u;\mu_F) T_{V(A)}^{(n)}(u; p_B^2, E_\gamma; \mu_F)\,. \tag{2.5}$$

This is a factorization formula expressing the correlation function as a convolution of genuinely nonperturbative and universal distribution amplitudes (DA) $\phi^{(n)}$ with process-dependent hard kernels $T^{(n)}$, to be calculated in perturbation theory; the overall factor $Q_u \chi \langle \bar{u}u\rangle$ ($Q_u = 2/3$) is pulled out for later convenience. In the above equation, n labels

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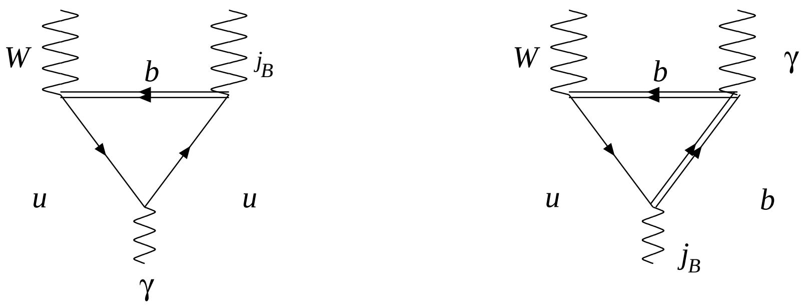

Figure 1: Diagrams with hard photon emission.

the twist of operators and μ_F is the factorization scale. The restriction on E_γ implies that $F_{A,V}^{\rm soft}$ cannot be calculated for all photon energies; to be specific, we restrict ourselves to $E_\gamma > 1\,{\rm GeV}$. The factorization formula holds if the infrared divergencies occurring in the calculation of $T^{(n)}$ are such that they can be absorbed into the universal distribution amplitudes $\phi^{(n)}$ and if the convolution integral converges. We find that this is indeed the case, at least for the leading twist-2 contribution and to first order in α_s. We also would like to stress that the above factorization formula has got nothing to do with the pQCD factorized expression for $F_{A,V}^{\rm hard}$ derived in [5, 6] (although we will discuss the relevance of our findings as compared to theirs in section 4) and that it is valid for arbitrary values of the b quark mass.

Let us now define the photon DAs that enter eq. (2.5). As mentioned above, we shall work in the background field gauge, which means that the expression (2.3), formulated for an outgoing photon, gets replaced by a correlation function of the time-ordered product of two currents in the vacuum, which is populated by an arbitrary EM field configuration $F_{\alpha\beta}$,

$$\Pi_F^\mu(p,q) = i\int d^4x e^{ipx}\frac{1}{\sqrt{4\pi\alpha}}\langle 0|T\{\bar u(x)\gamma^\mu(1-\gamma_5)b(x)\bar b(0)i\gamma_5 u(0)\}|0\rangle_F\,, \qquad (2.6)$$

where the subscript F indicates that an EM background field B_μ is included in the action. The calculation is explicitly gauge-invariant, and it is only in the final step that we select one specific field configuration corresponding to an outgoing photon with momentum q:

$$F_{\alpha\beta}(x) \to -i(\epsilon^*_\alpha q_\beta - \epsilon^*_\beta q_\alpha)e^{iqx}\,.$$

Following ref. [10], we define the leading-twist DA as the vacuum expectation value of the nonlocal quark-antiquark operator with light-like separations, in the EM background field configuration $F_{\alpha\beta}$:

$$\langle 0|\bar q(z)\sigma_{\alpha\beta}[z,-z]_F q(-z)|0\rangle_F = e_q\,\chi\,\langle\bar q q\rangle\int_0^1 du\, F_{\alpha\beta}(-\xi z)\phi_\gamma(u)\,, \qquad (2.7)$$

where $\xi = 2u-1$, $z^2=0$ and $[z,-z]_F$ is the path-ordered gauge-factor

$$[z,-z]_F = {\rm P}\exp\left\{i\int_0^1 dt\, 2z_\mu[gA^\mu((2t-1)z) + e_q B^\mu((2t-1)z)]\right\}, \qquad (2.8)$$

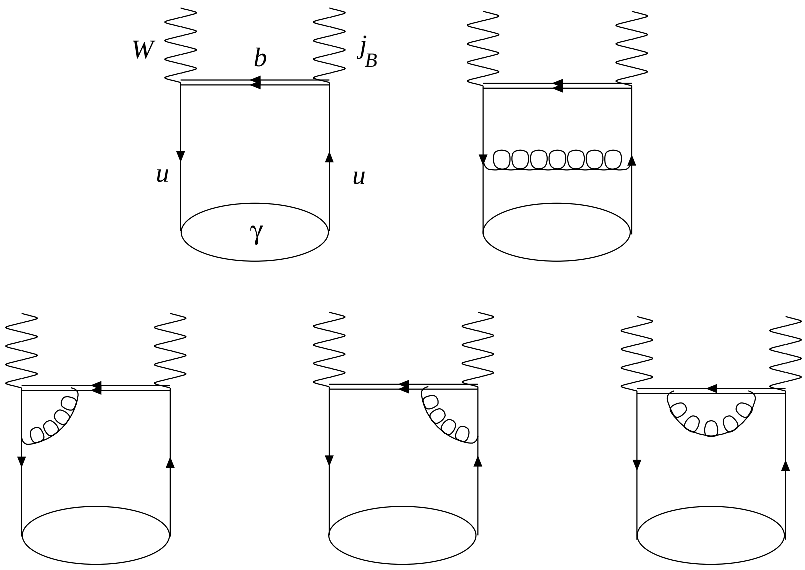

Figure 2: Leading twist-2 soft photon contributions to the correlation function (2.2).

including both the gluon field $A_\mu = A_\mu^a \lambda^a/2$ and the EM background field B_μ. e_q is the electric charge of the light quark q, e.g. $e_u = 2/3\sqrt{4\pi\alpha}$, $\langle \bar{q}q \rangle$ the quark condensate and χ its magnetic susceptibility — defined via the local matrix element $\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = e_q\,\chi\,\langle\bar{q}q\rangle\,F_{\alpha\beta}$, which implies the normalization $\int_0^1 du\phi_\gamma(u) = 1$. ϕ_γ can be expanded in terms of contributions of increasing conformal spin (cf. [15] for a detailed discussion of the conformal expansion of DAs):

$$\phi_\gamma(u,\mu) = 6u(1-u)\left[1 + \sum_{n=2,4,\dots}^{\infty} \phi_n(\mu) C_n^{3/2}(2u-1)\right], \tag{2.9}$$

where $C_n^{3/2}$ are Gegenbauer polynomials and $0 \le u \le 1$ is the collinear momentum fraction carried by the quark. The usefulness of the conformal expansion lies in the fact that the Gegenbauer moments $\phi_n(\mu)$ renormalize multiplicatively in LO perturbation theory. At higher twist, there exists a full plethora of photon DAs, which we refrain from defining in full detail, but refer the reader to the discussion in ref. [10], in particular section 4.1.

The task is now to calculate the hard kernels $T^{(n)}$, to $O(\alpha_s)$ in leading-twist and tree-level for higher twist. As for the former, the relevant diagrams are shown in figure 2. The diagrams are IR divergent, which divergence can be absorbed into the DA $\chi\langle\bar{u}u\rangle\phi_\gamma$; we have checked that this is indeed the case. A critical point in pQCD calculations involving heavy particles is the possibility of soft divergencies, which manifest themselves as divergence of the u integration at the endpoints; the success of the approach advocated in [1, 2] relies precisely on the fact that it yields a factorization formula for nonleptonic decays where these divergencies are absent in the heavy quark limit (but come back at order $1/m_b$). We find that there are no soft divergencies in our case,[2] which reiterates what has been

[2]Note that this statement applies to the *full* correlation function with finite m_b, not only to the heavy quark limit.

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found for other heavy-light transitions, cf. [16]–[18]. We thus confirm the factorization formula (2.5) to $O(\alpha_s)$ at twist-2. Explicit expressions for the spectral densities of the diagrams are too bulky to be given here; they can be obtained from the authors. At this point we only note that, to leading-twist accuracy, $\Pi_A \equiv \Pi_V$.

As for the higher-twist contributions, we obtain the following results:

$$\begin{aligned}
\Pi_A &= -Q_u f_{3\gamma}\int_0^1 du \frac{\bar{\psi}^{(v)}(u)}{s^2} m_b + \frac{1}{2} Q_u \langle \bar{u}u\rangle \int_0^1 du \frac{\bar{h}(u)}{s^2}\left[1+\frac{2m^2}{s}\right] - \\
&- Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{S(\underline{\alpha})}{\tilde{s}^2}(1-2v) + \frac{1}{6}Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{\tilde{S}(\underline{\alpha})}{\tilde{s}^2} - \\
&-2Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{p\cdot q}{\tilde{s}^3}\left[\bar{T}_1(\underline{\alpha})-(1-2v)\bar{T}_2(\underline{\alpha})+(1-2v)\bar{T}_3(\underline{\alpha})-\bar{T}_4(\underline{\alpha})\right], \quad (2.10)\\
\Pi_V &= \frac{1}{2}Q_u f_{3\gamma}\int_0^1 du \frac{\psi^{(a)}(u)}{s^2} m_b - \\
&- Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{S(\underline{\alpha})}{\tilde{s}^2} + \frac{1}{6}Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{\tilde{S}(\underline{\alpha})}{\tilde{s}^2}(1-2v) - \\
&- 2Q_u\langle \bar{u}u\rangle \int_0^1 dv \int \mathcal{D}\underline{\alpha}\frac{p\cdot q}{\tilde{s}^3}\left[\bar{T}_1(\underline{\alpha})-\bar{T}_2(\underline{\alpha})+(1-2v)\bar{T}_3(\underline{\alpha})-(1-2v)\bar{T}_4(\underline{\alpha})\right], \quad (2.11)
\end{aligned}$$

where $s = m_b^2 - (p+uq)^2$ and $\tilde{s} = m_b^2 - (p+\zeta q)^2$ with $\zeta = (\alpha_{\bar{q}} - \alpha_q + (2v-1)\alpha_g + 1)/2$. Definitions and explicit expressions for the higher twist DAs $\bar{\psi}^{(v)}(u)$ and $\psi^{(a)}(u)$ (two-particle twist-3), $\bar{h}(u)$ (two-particle twist-4), $S(\underline{\alpha})$, $\tilde{S}(\underline{\alpha})$ (three-particle twist-3) and $\bar{T}_i(\underline{\alpha})$ (three-particle twist-4) can be found in [10]. Note that in contrast to the leading-twist results, $\Pi_V^{\text{higher twist}} \neq \Pi_A^{\text{higher twist}}$.

3. Numerical results for $F_{A,V}^{\text{soft}}$

Before presenting numerical values for the soft part of the form factors, we first discuss the numerical input to the sum rule (2.4). As for the photon DA, we use the values and parametrizations derived in [10], notably for the normalization of the twist-2 matrix element (2.7):

$$(\chi\langle \bar{u}u\rangle)(1\,\text{GeV}) = -(0.050 \pm 0.015)\,\text{GeV}\,.$$

As discussed in [10], there is no conclusive evidence for ϕ_γ to differ significantly from its asymptotic form, so we set

$$\phi_\gamma(u) = 6u(1-u)\,.$$

The remaining hadronic matrix elements characterizing higher-twist DAs are detailed in [10]. Note that we evaluate scale-dependent quantities at the factorization scale $\mu_F^2 = m_B^2 - m_b^2$ [19]; the dependence of the form factors on μ_F is very small, as all numerically sizeable contributions are now available to NLO in QCD, which ensures good cancellation of the residual scale dependence.

As for the remaining parameters occurring in (2.4), we have the sum rule specific parameters M^2 and s_0, that is the Borel parameter and the continuum threshold, respectively. In addition, the sum rule depends on m_b, the b quark mass, and f_B, the leptonic decay constant of the B. f_B can in principle be measured from the decay $B \to \ell\bar{\nu}_\ell$, which, due to the expected smallness of its branching ratio, BR$\sim O(10^{-6})$, has, up to now, escaped experimental detection. f_B is one of the best-studied observables in lattice-simulations with heavy quarks; the current world-average from unquenched calculations with two dynamical quarks is $f_B = (200 \pm 30)\,\mathrm{MeV}$ [20]. It can also be calculated from QCD sum rules: the most recent determinations [21] include $O(\alpha_s^2)$ corrections and find $(206 \pm 20)\,\mathrm{MeV}$ and $(197 \pm 23)\,\mathrm{MeV}$, respectively. For consistency, we do not use these results, but replace f_B in (2.4) by its QCD sum rule to $O(\alpha_s)$ accuracy, including the dependence on s_0 and M^2, and use the corresponding "optimum" ranges of continuum threshold and Borel parameter also in evaluating the Borel-transformed correlation function $\Pi_{V(A)}$, i.e. the l.h.s. of (2.4). For the b quark mass, we use an average over recent determinations of the $\overline{\mathrm{MS}}$ mass, $\overline{m}_{b,\overline{\mathrm{MS}}}(\overline{m}_b) = (4.22 \pm 0.08)\,\mathrm{GeV}$ [22, 23], which corresponds to the one-loop pole-mass $m_{b,1\mathrm{L-pole}} = (4.60 \pm 0.09)\,\mathrm{GeV}$. With these values we find $f_B = (192 \pm 22)\,\mathrm{GeV}$ (the error only includes variation with m_b and M^2, at optimized s_0), in very good agreement with both lattice and QCD sum rules to $O(\alpha_s^2)$ accuracy. For $m_b = (4.51, 4.60, 4.69)\,\mathrm{GeV}$ the optimized s_0 are $(34.5, 34.0, 33.5)\,\mathrm{GeV}^2$, and the relevant range of M^2 is $(4.5 - -8)\,\mathrm{GeV}^2$.

In figure 3 we plot the different contributions to $F^{\mathrm{soft}}_{V,A}(0)$ as function of M^2, for $m_b = 4.6\,\mathrm{GeV}$, $s_0 = 34\,\mathrm{GeV}^2$ and the central value of $\chi\langle\bar{u}u\rangle$. It is evident that the sum rule is dominated by twist-2 contributions and that both radiative corrections and higher-twist terms are well under control. Note also the minimal sensitivity to M^2 which indicates a "well-behaved" sum rule. Varying m_b, s_0 and the other input parameters within the ranges specified above, we find

$$F^{\mathrm{soft}}_A(0) = 0.07 \pm 0.02\,, \qquad F^{\mathrm{soft}}_V(0) = 0.09 \pm 0.02\,. \tag{3.1}$$

As mentioned before, the above results are obtained using the asymptotic form of the twist-2 photon DA. Although there is presently no evidence for nonzero values of higher Gegenbauer moments, the ϕ_n in (2.9), it may be illustrative to estimate their possible impact on the form factors. As a guideline for numerics, we choose ϕ_2^γ to be equal to $\phi_2^{\rho\perp}$, its analogue for the transversely polarized ρ meson, as determined in [24]: $\phi_2^{\rho\perp}(1\,\mathrm{GeV}) = 0.2 \pm 0.1$. In figure 4 we plot the twist-2 contribution to the form factors obtained with the asymptotic ϕ_γ and the corrections induced by nonzero $\phi_2^\gamma(\mu_F)$. It is clear that the effect is moderate and at most about 20%. It is also interesting to note that positive values of ϕ_2^γ, which are in accordance with assuming ρ meson dominance for the photon, increase the form factor, which means that our results are likely to be an *underestimate* of the soft contributions rather than the contrary.

In figure 5, we show the dependence of $F^{\mathrm{soft}}_{A,V}(p^2)$ on the momentum transfer p^2, including the variation of all input parameters in their respective ranges. The form factors can be fitted by the following formula:

$$F^{\mathrm{soft}}_{A(V)}(p^2) = \frac{F^{\mathrm{soft}}_{A(V)}(0)}{1 - a_{A(V)}\left(p^2/m_B^2\right) + b_{A(V)}\left(p^2/m_B^2\right)^2}\,. \tag{3.2}$$

The fit parameters $F^{\mathrm{soft}}_{A(V)}(0)$, $a_{A(V)}$ and $b_{A(V)}$ for each curve in the figure are given in table 1. The above formula fits the full sum rule results to 1% accuracy for $0 < p^2 < 17\,\mathrm{GeV}^2$, which

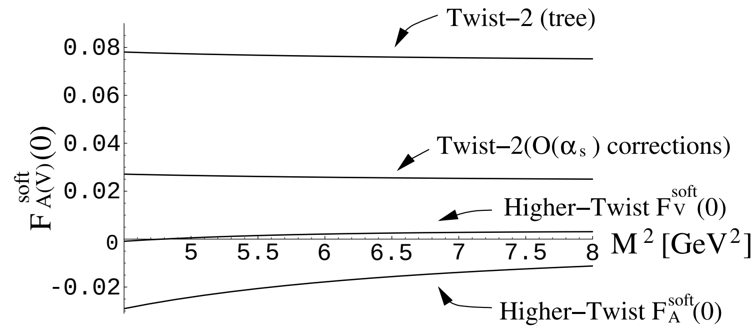

Figure 3: The soft photon contributions to $F_{V(A)}(0)$ in dependence of the Borel parameter M^2. The twist-2 contributions to both form factors are equal. The plot refers to $m_b = 4.6\,\text{GeV}$ and $s_0 = 34\,\text{GeV}^2$.

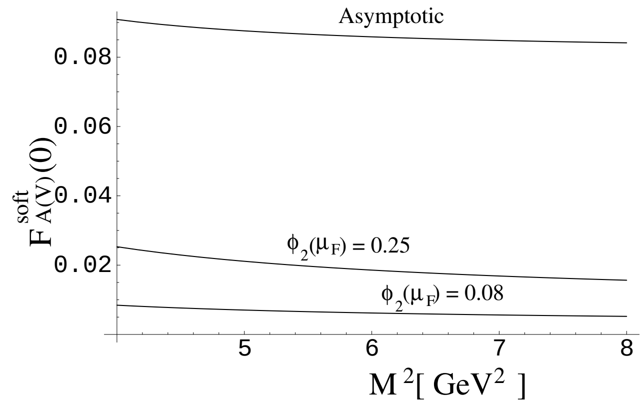

Figure 4: Contributions to $F^{\text{soft}}_{V(A)}$ from different values ϕ_2 of the 2nd Gegenbauer moment of the photon DA ($m_b = 4.6\,\text{GeV}$, $s_0 = 34\,\text{GeV}^2$).

corresponds to $1\,\text{GeV} < E_\gamma < m_B/2$. Note that the uncertainty induced by M^2 and s_0 is very small: up to 5% for F_V and up to 10% for F_A. The main theoretical uncertainty comes from $\chi\langle\bar{q}q\rangle$.

4. Calculation of λ_B — comparison to pQCD

As mentioned above, the $B \to \gamma$ form factors have also been calculated in the framework of pQCD factorization [7, 5, 6]. This approach employs the limit $m_b \to \infty$ (HQL), in which the soft contributions vanish. The form factors $F^{\text{hard}}_{A(V),\text{HQL}}$ are equal and at tree level given by

$$F^{\text{hard}}_{A,\text{HQL}}(E_\gamma) \equiv F^{\text{hard}}_{V,\text{HQL}}(E_\gamma) = \frac{f_B m_B Q_u}{2\sqrt{2}E_\gamma} \int_0^\infty dk_+ \, \frac{\Phi^B_+(k_+)}{k_+} =: \frac{f_B m_B Q_u}{2E_\gamma}\,\frac{1}{\lambda_B}\,, \tag{4.1}$$

where the light-cone DA Φ^B_+ of the B meson depends on the momentum of the light spectator quark, k_+. The calculation of radiative corrections to this formula has been the

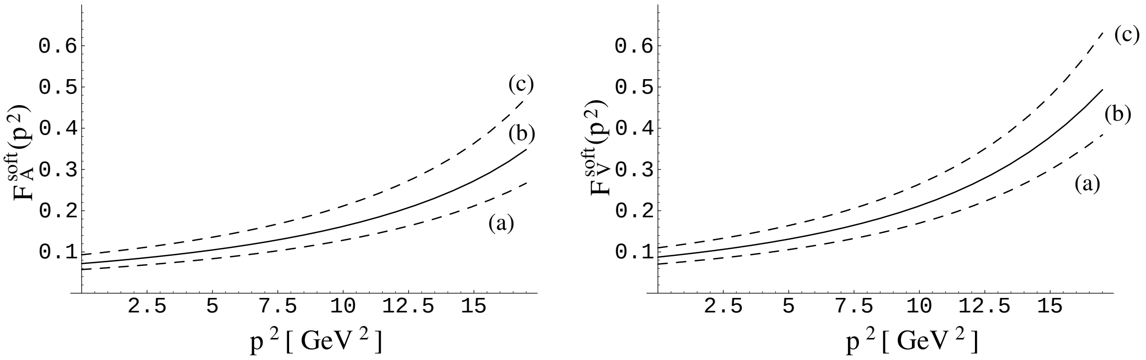

Figure 5: Dependence of $F_{V(A)}^{\rm soft}$ on the momentum transfer p^2. Solid line: light-cone sum rule for central values of input parameters. Dashed lines: spread of $F_{V(A)}^{\rm soft}$ upon variation of input parameters. The exact parameter sets for the curves (a) to (c) are listed in table 1. Note that the uncertainty originating from M^2 and s_0 is less than 10%.

Input parameters	(a)	(b)	(c)
m_b [GeV]	4.69	4.60	4.51
s_0 [GeV^2]	33.5	34.0	34.5
M^2 [GeV^2]	8	6	5
$\chi\langle\bar{u}u\rangle(\mu = 1\,\text{GeV})$ [GeV]	-0.035	-0.050	-0.065
Fit parameters	(a)	(b)	(c)
$F_A^{\rm soft}(0)$	0.057	0.072	0.093
a_A	1.97	1.97	1.95
b_A	1.18	1.15	1.08
$F_V^{\rm soft}(0)$	0.071	0.088	0.110
a_V	2.09	2.08	2.05
b_V	1.25	1.22	1.16

Table 1: Input parameter sets for figure 5 and fit parameters for eq. (3.2).

subject of a certain controversy, cf. refs. [5]–[7]. The parameter λ_B does not scale with m_b in the HQL and hence is of natural size $O(\Lambda_{\rm QCD})$; it has been quoted as $\lambda_B = 0.3\,\text{GeV}$ [1] and $\lambda_B = (0.35{\pm}0.15)\,\text{GeV}$ [5], but without calculation. Although presently any statement about the numerical size of λ_B appears slightly precarious, since Φ_B^+ and hence λ_B depend in a yet unknown way on the factorization scale μ_F, we nonetheless venture to present the first calculation of λ_B not based on any specific parametrization of Φ_B^+.[3] To that purpose, we recall that the hard contribution to $F_{A,V}$ can be obtained, in the QCD sum rule approach, from the local contributions to the correlation function $\Pi_{A,V}$, Eq. (2.2), which, to leading order in perturbation theory, correspond to the diagrams shown in figure 1 and have been calculated in [8]. In order to extract λ_B via (4.1) from the local QCD sum rule for $F_{A,V}^{\rm hard}$, we first have to find its HQL. QCD sum rules in the heavy quark limit have actually been studied in quite some detail, cf. [26], with the following result for the scaling relations of

[3] In ref. [25], a specific parametrization of Φ_B^+ was obtained, motivated by QCD sum rules, which allows one to calculate λ_B in terms of $\bar{\Lambda}$. One obtains $\lambda_B = 2/3\,\bar{\Lambda} \approx 0.47\,\text{GeV}$, which is smaller than our estimate.

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the sum rule specific parameters M^2 and s_0:

$$M^2 \to 2m_b\tau\,, \qquad s_0 \to m_b^2 + 2m_b\omega_0\,. \tag{4.2}$$

Applying these relations to the sum rule for $F_{V,A}^{\rm hard}$ and using (4.1), we obtain the following expression:

$$e^{-\bar\Lambda/\tau}\,\frac{f_B^2 m_B^2}{m_b E_\gamma}\,\frac{1}{\lambda_B} = \frac{3}{\pi^2 E_\gamma}\int_0^{\omega_0} d\omega\,\omega\,e^{-\omega/\tau}\,. \tag{4.3}$$

We note that the factor $1/E_\gamma$ on the r.h.s. arises automatically in the HQL of the correlation function. $\bar\Lambda$ is the binding energy of the b quark in the B meson, $\bar\Lambda = m_B - m_b$. On the l.h.s. of (4.3), the expression $f_B^2 m_B^2/m_b$ still contains $1/m_b$ corrections; in the rigorous HQL we have $f_B^2 m_B^2/m_b \to f_{\rm stat}^2$. $f_{\rm stat}$ is known both from lattice calculations and QCD sum rules; in the same spirit that we applied for calculating $F_{A,V}^{\rm soft}$, we replace, for the numerical evaluation of λ_B, $f_{\rm stat}^2$ by its sum rule [26]:

$$f_{\rm stat}^2 e^{-\bar\Lambda/\tau} = \frac{3}{\pi^2}\int_0^{\omega_0} d\omega\,\omega^2 e^{-\omega/\tau}\,, \tag{4.4}$$

where we have suppressed (small) condensate contributions. Combining (4.3) and (4.4), we find

$$\lambda_B = \frac{\int_0^{\omega_0} d\omega\,\omega^2 e^{-\omega/\tau}}{\int_0^{\omega_0} d\omega\,\omega e^{-\omega/\tau}}\,. \tag{4.5}$$

This is our sum rule for λ_B, which, admittedly, is the first rather than the last word in the story of how to calculate λ_B. It can and should be improved by including both radiative (which will also settle the issue of scale dependence) and nonperturbative corrections. For fixing the sum rule parameters ω_0 and τ, we exploit the fact that $\bar\Lambda = m_B - m_b \approx 0.7\,\text{GeV}$ is known; the corresponding sum rule reads

$$\bar\Lambda = \frac{\int_0^{\omega_0} d\omega\,\omega^3 e^{-\omega/\tau}}{\int_0^{\omega_0} d\omega\,\omega^2 e^{-\omega/\tau}}\,, \tag{4.6}$$

which can be derived from (4.4) by taking one derivative in $1/\tau$. None of the sum rules (4.3)–(4.5) is "well-behaved" in the sense that they feature no stability plateau in τ, since the perturbative term is not counterbalanced by a nonperturbative one. Nonetheless, taking $0.5\,\text{GeV} < \tau < 1\,\text{GeV}$ as indicated by our preferred range of M^2 used above and the scaling laws (4.2), we find $\omega_0 = 1\,\text{GeV}$ and then, from (4.5),

$$\lambda_B \approx 0.57\,\text{GeV}\,.$$

Given the neglect of $O(\alpha_s)$ corrections and nonperturbative terms, it is difficult to attribute an error to that number. For this reason, we check if our result is compatible with the local duality approximation, which corresponds to the limit $\tau \to \infty$. From (4.3) and (4.4) we then obtain the interesting relation

$$\lambda_B = \frac{8}{9}\,\bar\Lambda,$$

which for $m_b = 4.6\,\mathrm{GeV}$ implies $\lambda_B = 0.6\,\mathrm{GeV}$. We thus conclude that the value of λ_B is set by $\bar{\Lambda}$ rather than $\Lambda_{\rm QCD}$ and depends strongly on the actual value of m_b; at present, nothing meaningful can be said about the error associated with λ_B and we thus quote as our final result

$$\lambda_B = 0.6\,\mathrm{GeV}\,. \tag{4.7}$$

We are now in a position to compare the numerical size of the pQCD result $F^{\rm hard}_{A(V),\rm HQL}$ to $F^{\rm soft}_{A(V)}$. At tree level, we have $F^{\rm hard}_{A(V),\rm HQL}(E_\gamma = m_B/2) = 0.22$, according to (4.1). Notwithstanding the additional hadronic uncertainties involved at NLO in QCD, we employ the models for Φ_B^+ advocated in [5] to obtain $F^{{\rm hard},NLO}_{A(V),\rm HQL}(E_\gamma = m_B/2) = 0.21 \pm 0.01$ for $\mu_F^2 = m_B^2 - m_b^2$, our choice of the factorization scale. This number has to be compared to (3.1), the soft contributions. We conclude that $F^{\rm soft}_{A,V}/F^{\rm hard}_{A(V),\rm HQL} \approx 0.3$ at maximum photon energy so that the parametrical scaling $F^{\rm soft}_{A,V} \sim O(1/m_b)$ is numerically relaxed. This number has to be interpreted with care as it relies on a tree-level estimate of λ_B. We nevertheless take the view that there is sufficient evidence to suggest that power-suppressed corrections to pQCD factorization in $B \to \gamma e\nu$ are numerically relevant.

5. Summary and conclusions

The relevance of B physics for extracting information on weak interaction parameters and new physics is limited by our lack of knowledge on nonperturbative QCD. Recent progress in describing the notoriously difficult nonleptonic decays in perturbative QCD factorization has raised hopes that a sufficiently accurate solution to the problem is around the corner. As much as this is a highly desirable goal, it is nonetheless necessary to critically examine the theoretical uncertainty of the method, which, at least at present, is set by the restriction to the heavy quark limit. A direct *theoretical* test of pQCD factorization in nonleptonic decays is currently not feasible, and any significant *experimental* deviation of measured decay rates or CP asymmetries from pQCD predictions is as likely to be attributed to new physics effects as to uncertainties in the predictions themselves. An indirect theoretical test becomes however possible in the admittedly phenomenologically not very attractive channel $B \to \gamma e\nu$, where alternative methods of calculation exist and allow one to assess effects suppressed by powers of m_b. In this letter, we have calculated corrections to the hard pQCD form factors, which are parametrically suppressed by one power of m_b. These "soft" corrections are induced by photon emission at large distances and involve the hadronic structure of the photon. We have also calculated λ_B, the first negative moment of the B meson distribution amplitude, a very relevant parameter for pQCD calculations. The calculation is admittedly rather crude, but amenable to improvement. Comparing the numerical size of the pQCD result to the soft contributions, we found that the latter are indeed sizeable, $O(30\%)$. This result implies an immediate caveat for pQCD analyses involving photon emission, in particular $B \to K^*\gamma$ and $B \to \rho\gamma$, e.g. [11, 12]. In a wider sense, it also adds a possible questionmark to pQCD analyses of purely hadronic B decays and emphasizes the relevance of power-suppressed corrections to the heavy quark limit.

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